Time-varying Beta in Functional Factor Models: Evidence from China

Lajos Horváth[a], Bo Li[b], Hemei Li[c], Zhenya Liu[c,d]

[a]*Department of Mathematics, University of Utah, Salt Lake City, UT 841120090, USA*
[b]*Bussiness School, Beijing International Studies University, Beijing, 100024, China*
[c]*School of Finance, Renmin University of China, Beijing, 100872, China*
[d]*CERGAM, Aix-Marseille University, 13090 Aix-en-Provence Cedex 02, France*

Abstract

This paper introduces the functional factor models with the time-varying beta. The advantage of doing this is that functional factor models give the time-varying beta intuitively, from which we find that in the Chinese A-share market, with both the Fama-French 3-factor model and the 5-factor model, the market factor has a positive effect on excess returns of A shares all the time, the size factor and the value factor have a positive impact on excess returns of A shares in a stable period, the investment factor had a positive effect after the non-tradable share reform and has changed to a negative impact since the 2008 financial crisis, while the profitability factor always has a negative impact on A shares.

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Email address: `hemei.li@ruc.edu.cn` (Hemei Li)

1. Introduction

Based on the mean-variance model (Markowitz, 1952), Sharpe (1964), Lintner (1965) and Mossin (1966) put forward the capital asset pricing model (CAPM), making the regression coefficient beta to measure exposure to market risk. Later, more and more scholars add firm-specific (unsystematic) risk factors into the CAPM, leading to the multi-factor models (Fama and French, 1992, 1993, 2015; Carhart, 1997; Acharya and Pedersen, 2005). However, most factor models are restricted to constant beta assumption, and existing researches about the time-varying beta mainly use the kalman filter model (Hameed, 1997; Zhou, 2013). In this paper, we propose a functional approach (Ramsay and Silverman, 2005; Horváth and Kokoszka, 2012) to dynamic asset pricing models based on the CAPM, the Fama-French three factor model and the five factor model, which can explore the time variation of exposures to risk factors.

There has always been controversy about the model setting and statistical testing of these factor models, especially for the beta. As the beta is unobservable, one simple estimation method is to perform linear regression with time series data, and then get the constant beta. Jayasinghe et al. (2014); Bu et al. (2019) argue that ordinary linear regression for constant betas may not be compatible with financial theory. In addition, we should notice that the constant beta assumption depends on stationarity, but whether or not the beta is stationary enough to behave like a constant. Levy (1971) studies the stationarity of betas with different portfolios and finds that betas fail to be stationary for the smaller portfolios. Further, Blume (1975) proves that betas exhibit mean-reversion tendency. Bos and Newbold (1984); Bollerslev et al. (1988) also show that beta is time-varying. Other than these, many scholars argue that asset returns are unstable, so they cannot be in favor of time-invariant betas (Campbell and Hentschel, 1992; Glosten et al., 1993). For statistical testing, Fama and French (2016) point out that the constant-beta assumption may be a potential problem in tests of asset pricing models. Bodurtha Jr and Mark (1991) propose the GMM test for the CAPM with time-varying returns and risks, and Velu and

Zhou (1999) extend the GMM test to multi-factor asset pricing models.

In view of the considerable evidence showing that estimated betas are significantly time-varying, asset pricing researches have been devoted to studying the time variation of betas. Merton (1973) and Breeden (1979) extend the one-period asset pricing model into multi-period models, and put forward the intertemporal capital asset pricing model and the consumption capital asset pricing model respectively. There are also many advocating conditional asset pricing models, such as the conditional CAPM (Ferson and Harvey, 1991; Jagannathan and Wang, 1996) and the dynamic conditional betas model (Engle, 2016), which are applied in equity, bond, and portfolios. In empirical research, the ARMA model, GARCH model, and Kalman filter model are mainly used to study the time variation of betas. Blume (1975); Sunder (1980) confirm the time-varying of betas with the AR (1) model and Ohlson and Rosenberg (1982); Collins et al. (1987) use the ARMA model. Ng (1991); Lee et al. (2001) analyze the time-varying volatility with various GARCH models. As for Kalman filter, Kim et al. (2009); Borup (2019) make the time-varying coefficient approach with Kalman filter technique. In addition, Kim and Kim (2016) reject the constant-volatility assumption and employ the nonparametric kernel method for time-varying volatility. Li et al. (2015) put forward a time-varying coefficient model with sieve approximation approach.

Functional analysis is widely used in quantum mechanics, bioengineering, and other fields. Recently, functional analysis is beginning to be used in finance. Ramsay and Silverman (2005), Ramsay and Silverman (2007) systematically introduce functional data analysis. Horváth and Kokoszka (2012) develop functional data analysis in time series data. Based on this theory, Kokoszka et al. (2015) introduce functional regression into the asset pricing model by transforming daily asset returns and the market factor into functions and constructing a functional factor model with both scalar factors and functional factors. However, Kokoszka et al. (2015) employ the functional factor model with constant betas assumption. In addition, Kokoszka et al. (2018) put forward a statistical significance test for risk factors in functional regression with functional cross-section returns and scalar risk factors. Also based on the research of Horváth and Kokoszka (2012), Cao et al. (2019) decompose the cross-section returns with functional principal component analysis (FPCA) and find the momentum and disposition effects in Chinas A-share market.

Motivated by the functional factor model shown in Kokoszka et al. (2015, 2018), we introduce functional asset pricing models, which can intuitively act out how betas change over time. The main contribution of this paper is building functional asset pricing models that give time-varying betas. One advantage of functional regression is that it allows the estimation of functional coefficients. Different from the model of Kokoszka et al. (2018), we make time-series data as functions of time t and both excess returns and risk factors are functional. Based on the functional factor models in this paper, we can test the validity of factor models without the restriction of constant betas.

The rest of this paper is organized as follows: Section 2 provides the functional factor models. Section 3 introduces the computational issues for time-varying beta and confidence intervals. Based on functional data illustrated in Section 4, we investigate the time variation of betas in Section 5. Section 6 summarises the application of functional factor models in the Chinese stock market.

2. Conventional and Functional Factor Models

2.1. Conventional Factor Models

For any security or portfolio, the conventional CAPM [1] is designed by the linear regression:

$$R_t = \alpha + \beta R_{M,t} + \varepsilon_t \tag{1}$$

where R_t is the excess return on any security or portfolio at time t[2]; $R_{M,t}$ is the excess return on market portfolio related to the modern portfolio theory,

[1] For any security or portfolio s, the conventional CAPM can be expressed as $R_t^s = \alpha^s + \beta^s R_{M,t} + \varepsilon_t^s$, to be concise, we omit superscript s.

[2] In this paper, we choose monthly data from January 1997 to December 2018 (264 months in total), so $t = 1, 2, \ldots, 264$.

representing the market factor.

The CAPM is gradually developed to the three-factor model (Fama and French, 1993):

$$R_t = \alpha + \beta_1 R_{M,t} + \beta_2 SMB_t + \beta_3 HML_t + \varepsilon_t \tag{2}$$

where SMB_t is the difference between returns on a value-weighted portfolio of small stocks and that of big stocks, representing the size factor; HML_t is the difference of returns for high and low book-to-market ratio, representing the value factor.

And the five-factor model (Fama and French, 2015):

$$R_t = \alpha + \beta_1 R_{M,t} + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 RMW_t + \beta_5 CMA_t + \varepsilon_t \tag{3}$$

where RMW_t is the difference of returns for robust and weak operating profit, representing the profitability factor; CMA_t is the difference of returns for conservative and aggressive investment, representing the investment factor.

2.2. Functional Factor Models

Now we introduce the functional analysis for factor models. By analogy with the conventional factor model, the functional CAPM [3] is:

$$R_i(t) = \alpha(t) + \beta(t) R_{M,i}(t) + \varepsilon_i(t) \tag{4}$$

where $R_i(t)$ is the functional time-series excess return on portfolio over time t [4]; $R_{M,i}(t)$ is the functional time-series market factor over time t [5]; $\beta(t)$ is the functional coefficient which can intuitively express the time-varying of beta.

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[3] For any portfolio s, the functional CAPM can be expressed as $R_i^s(t) = \alpha^s(t) + \beta^s(t) R_{M,i}(t) + \varepsilon_i^s(t)$, where functional observation i represents the finer portfolio or stock in portfolio s, to be concise, we omit superscript s.

[4] We make excess return of stock or portfolio i a function of time t ($t = 1, 2, \ldots, 264$), i.e., $R_i(t)$.

[5] We make market factor a function of time t ($t = 1, 2, \ldots, 264$), i.e., $R_{M,i}(t)$, and for different functional excess returns of stocks or portfolios, there is the same functional market factor, that is, $R_{M,i}(t)$ stays the same for different i. So do other risk factors.

The functional three-factor model is:

$$R_i(t) = \alpha(t) + \beta_1(t)R_{M,i}(t) + \beta_2(t)SMB_i(t) + \beta_3(t)HML_i(t) + \varepsilon_i(t) \quad (5)$$

where $SMB_i(t)$ is the functional time-series size factor, and $HML_i(t)$ is the functional time-series value factor.

The functional five-factor model is:

$$R_i(t) = \alpha(t)+\beta_1(t)R_{M,i}(t)+\beta_2(t)SMB_i(t)+\beta_3(t)HML_i(t)+\beta_4(t)RMW_i(t)+\beta_5(t)CMA_i(t)+\varepsilon_i(t) \quad (6)$$

where $RMW_i(t)$ is the functional time-series profitability factor, and $CMA_i(t)$ is the functional time-series investment factor.

3. Methodology

In this section, we introduce the computational issues for time-varying beta. According to the characteristics of functional regression, we can set beta as a function to study the time-varying.

The general functional factor regression can be expressed as follows:

$$R_i(t) = \sum_{j=1}^{m} \beta_j(t)F_{i,j}(t) + \varepsilon_i(t) \quad (7)$$

or

$$\boldsymbol{R}(t) = \boldsymbol{\beta}(t)\boldsymbol{F}(t) + \boldsymbol{\varepsilon}(t) \quad (8)$$

where $\boldsymbol{\beta}(t) = (\beta_1(t), \ldots, \beta_m(t))$ and $\boldsymbol{F}(t) = (\boldsymbol{F}_1(t), \ldots, \boldsymbol{F}_m(t))$ contains all functional time-series factors.

As the model is a standard general linear model, we can choose $\boldsymbol{\beta}(t)$ by the standard least squares criterion. We extend the method of minimising the residual sum of squares to the functional regression case, and the least squares fitting criterion can be shown as:

$$\mathrm{LMSSE}(\boldsymbol{\beta}) = \int [\boldsymbol{R}(t) - \boldsymbol{\beta}(t)\boldsymbol{F}(t)]'[\boldsymbol{R}(t) - \boldsymbol{\beta}(t)\boldsymbol{F}(t)]dt \quad (9)$$

The functional excess return $R_i(t)$ is expressed by

$$R_i(t) = \sum_{k=1}^{K} c_{i,k}\phi_k(t) = \boldsymbol{C}_i'\boldsymbol{\Phi}(t) \tag{10}$$

Similarly, to obtain the time-varying beta, we should define time-series curve-value $\beta_j(t)(t = 1, 2, \ldots, m)$, which is the main point in our functional factor model. The functional beta $\beta_j(t)$ is expressed by

$$\beta_j(t) = \sum_{k=1}^{K_j} b_{j,k}\psi_{j,k}(t) = \boldsymbol{b}_j'\boldsymbol{\psi}_j(t) \tag{11}$$

where $\boldsymbol{b}_j$ is the coefficient vector for functional beta $\beta_j(t)$ and $\boldsymbol{\psi}_j(t)$ is the Fourier basis functions. So far, the problem of estimating beta $\beta_j(t)$ has turned into valuating coefficient vector $\boldsymbol{b}_j$.

To express the model explicitly, we define $\boldsymbol{B} = (\boldsymbol{b}_1', \boldsymbol{b}_2', \ldots, \boldsymbol{b}_m')$ and

$$\boldsymbol{\Psi}(t) = \begin{pmatrix} \boldsymbol{\psi}_1(t) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{\psi}_m(t) \end{pmatrix}.$$

Then the functional factor model can be rewritten as

$$\boldsymbol{R}(t) = \boldsymbol{B}\boldsymbol{\Psi}(t)\boldsymbol{F}(t) + \boldsymbol{\varepsilon}(t) \tag{12}$$

and the least squares fitting criterion is:

$$\begin{aligned} \text{LMSSE}(\boldsymbol{\beta}) &= \int [\boldsymbol{R}(t) - \boldsymbol{B}\boldsymbol{\Psi}(t)\boldsymbol{F}(t)]'[\boldsymbol{R}(t) - \boldsymbol{B}\boldsymbol{\Psi}(t)\boldsymbol{F}(t)]dt \\ &= \int \boldsymbol{R}'(t)\boldsymbol{R}(t) - 2\boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{B}'\boldsymbol{R}(t) + \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{B}'\boldsymbol{B}\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt \end{aligned} \tag{13}$$

Based on (13), we can find the coefficient matrix B is the solution of

$$\boldsymbol{B}\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt = \int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{R}(t)dt \tag{14}$$

Therefore, we can evaluate the coefficient matrix B, and spontaneously get the functional time-series betas $\boldsymbol{\beta}(t)$, which intuitively shows the time variation.

Next, we will determine the confidence intervals. After figuring out the estimated coefficient matrix $\hat{B}$, we change the problem of computing confidence intervals (or standard deviation) of betas into calculating the standard deviation of coefficient vectors.

The coefficient matrix of betas can be expressed by

$$\begin{aligned}\hat{\boldsymbol{B}} &= \left(\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt\right)^{-1}\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{R}(t)dt \\ &= \left(\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt\right)^{-1}\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{C}\boldsymbol{\Phi}(t)dt \qquad (15)\end{aligned}$$

Then we get[6]

$$\text{vec}(\hat{\boldsymbol{B}}) = \left(\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt\right)^{-1}\int \boldsymbol{\Phi}'(t)\otimes \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)dt\text{vec}(\boldsymbol{C}) \qquad (16)$$

We denote $\boldsymbol{S}_\psi$ mapping raw data of excess return $\boldsymbol{R}$ into coefficient matrix $\boldsymbol{C}$, that is, $\boldsymbol{C} = \boldsymbol{R}\boldsymbol{S}_\psi$, then

$$\text{vec}(\boldsymbol{C}) = (\boldsymbol{S}'_\psi(t)\otimes \boldsymbol{I})\text{vec}(\boldsymbol{R}) \qquad (17)$$

The variance of raw data $\boldsymbol{R}$ is given by

$$\text{var}(\text{vec}(\boldsymbol{R})) = \Sigma_\varepsilon \otimes \boldsymbol{I} \qquad (18)$$

where Σ_ε is the variance-covariance matrix of residual vectors. Then the variance of estimated coefficient matrix $\hat{\boldsymbol{B}}$ is

$$\text{var}(\text{vec}(\hat{\boldsymbol{B}})) = \boldsymbol{A}(\boldsymbol{S}'_\psi(t)\otimes \boldsymbol{I})\Sigma_\varepsilon \otimes \boldsymbol{I}(\boldsymbol{S}'_\psi(t)\otimes \boldsymbol{I})'\boldsymbol{A}' \qquad (19)$$

[6] vec-operator: $\text{vec}(\boldsymbol{A}) = (a_{11}, a_{21}, \ldots, a_{m1}, \ldots, a_{1n}, a_{2n}, \ldots, a_{mn})'$ where $\boldsymbol{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn}\end{pmatrix}$.

Kronecker-operator $\otimes$: there are matrices $\boldsymbol{A}_{m\times n}$ and $\boldsymbol{B}_{p\times q}$, then we can get a $mn\times pq$ matrix as follows:

$$\boldsymbol{A}\otimes\boldsymbol{B} = \begin{pmatrix} a_{11}\boldsymbol{B} & \cdots & a_{1n}\boldsymbol{B} \\ a_{21}\boldsymbol{B} & \cdots & a_{2n}\boldsymbol{B} \\ \vdots & \ddots & \vdots \\ a_{m1}\boldsymbol{B} & \cdots & a_{mn}\boldsymbol{B}\end{pmatrix}$$

where $\boldsymbol{A} = \left(\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt\right)^{-1} \int \boldsymbol{\Phi}'(t) \otimes \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)dt$. So far, we have the variance of the estimated coefficient matrix $\hat{\boldsymbol{B}}$, and then we can compute the confidence intervals of estimated time-varying betas.

4. Data

In this paper, we intend to investigate functional factor models in the Chinese stock market. We choose all A shares (except stocks denoted ST and *ST [7]) returns with monthly frequency from the CSMAR database and all risk factors data with monthly frequency from the China Asset Management Research Center of CUFE. The data period is from January 1997 to December 2018 (264 months in total).

We construct 25 (or 100) value-weighted portfolios as observations, and different observations correspond to different excess returns (dependent variable) and the same risk factors (independent variables):

Dependent variable: excess returns on 25 (or 100) Size-B/M portfolios. [8] We sort all A shares by the market value and divide them into five (or ten) groups, then sort the stocks in each size group by the book-to-market ratio and divide them into finer five (or ten) groups, so that eventually, we have 25 Size-B/M portfolios (or 100 Size-B/M portfolios). The excess returns on value-weighted portfolios are shown as follows:

$$R_{i,t} = \sum_k \frac{mv_{k,t}^i}{\sum_k mv_{k,t}^i} r_{k,t}^i, \quad i = 1, 2, \dots, 25;\ t = 1, 2, \dots, 264 \tag{20}$$

or

$$R_{i,t} = \sum_k \frac{mv_{k,t}^i}{\sum_k mv_{k,t}^i} r_{k,t}^i, \quad i = 1, 2, \dots, 100;\ t = 1, 2, \dots, 264 \tag{21}$$

[7] In the Chinese stock market, stocks with ST denotes that the listed company has been in deficit for two consecutive fiscal years, and stocks with *ST denotes that the listed company has been in deficit for three consecutive fiscal years, both of these companies are faced with delisting risk.

[8] In the robust test, we also construct 25 (or 100) Size-OP Portfolios and 25 (or 100) Size-Inv Portfolios, where OP represents operating profitability and Inv represents investment.

where $R_{i,t}$ is the excess return on portfolio i at time t, $r_{k,t}^i$ is the excess return on stock k at time t in ith portfolio and $mv_{k,t}^i$ is the corresponding market value for stock k.

Independent variables: risk factors. For different observations (25 or 100 portfolios), we apply the same (25 or 100) risk factors (market factor, size factor, value factor, profitability factor and investment factor) constructed according to the method in Fama and French (2015).

Next, we build functional data for time-series excess returns and risk factors. To specification, we map the time-series data $\{R_{i,t}\}_{t=1}^{264}$ into $L^2[0,1]$ to generate functional data $\{R_i(t), 0 \leq t \leq 1\}$, and $R_i(t)$ can be expressed by basis function expansion Horváth and Kokoszka (2012):

$$R_i(t) = \sum_{k=1}^{K} c_{i,k}\phi_k(t) = \boldsymbol{C}_i'\boldsymbol{\Phi}(t) \tag{22}$$

where $\boldsymbol{\Phi}(t)$ is the basis functions, and $\boldsymbol{C}_i$ is a coefficient vector of the basis function expansion, then the general functional factor model is as follows:

$$R_i(t) = \sum_{j=1}^{m} \beta_j(t)\overline{F_{i,j}}(t) + \varepsilon_i(t), \quad i = 1, 2, \ldots, 25 \tag{23}$$

or

$$R_i(t) = \sum_{j=1}^{m} \beta_j(t)\overline{F_{i,j}}(t) + \varepsilon_i(t), \quad i = 1, 2, \ldots, 100. \tag{24}$$

where $\overline{F_{i,j}}(t)$ denotes every functional risk factor $F_{i,j}(t)$ is the same for all i. Generally speaking, there are two kinds of basis functions we can choose: one is the Fourier basis functions which are suitable for periodic observations, and the other is the spline basis functions which are ideal for non-periodic observations. We choose the Fourier basis functions because we think both excess returns and risk factors are periodic in the long run, so do time-varying betas. The raw data of factors and functional factors are shown in Figure1-Figure5, and the Fourier basis functions for estimated betas are shown in Figure6.

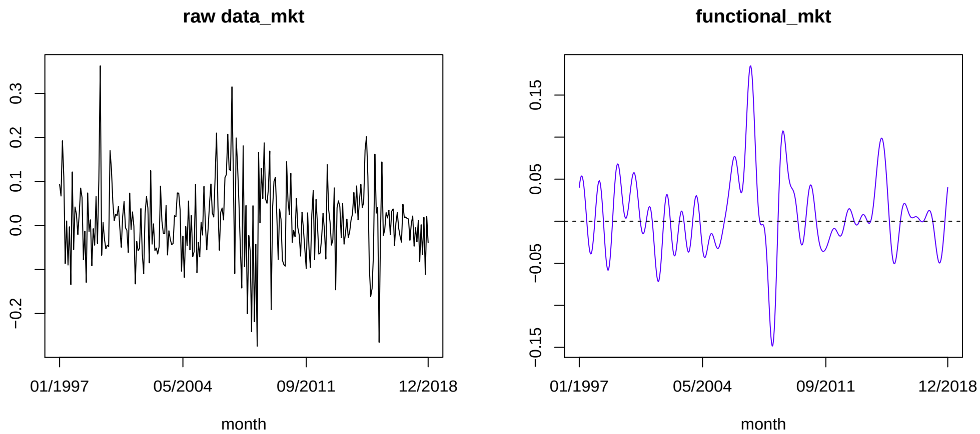

Figure 1: The market factor in the Chinese stock market (January 1997 - December 2018). The left panel shows raw data of the market factor, and the right panel shows the functional market factor.

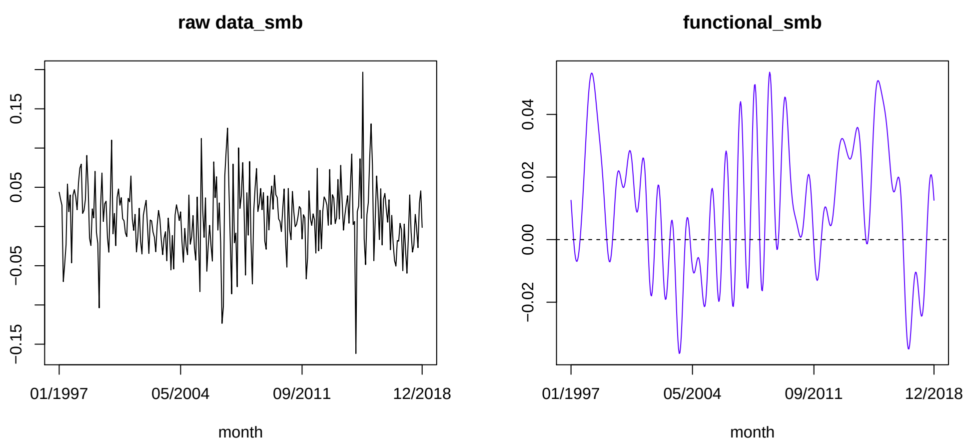

Figure 2: The size factor in the Chinese stock market (January 1997 - December 2018). The left panel shows raw data of the size factor, and the right panel shows the functional size factor.

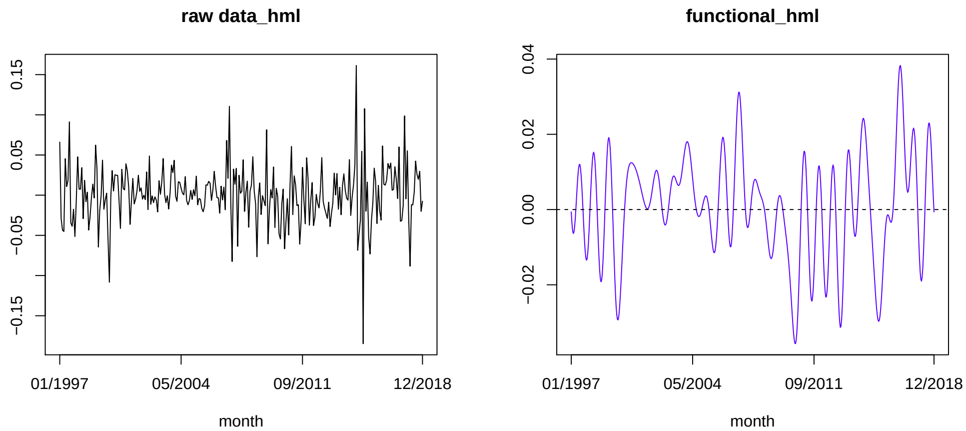

Figure 3: The value factor in the Chinese stock market (January 1997 - December 2018). The left panel shows raw data of the value factor, and the right panel shows the functional value factor.

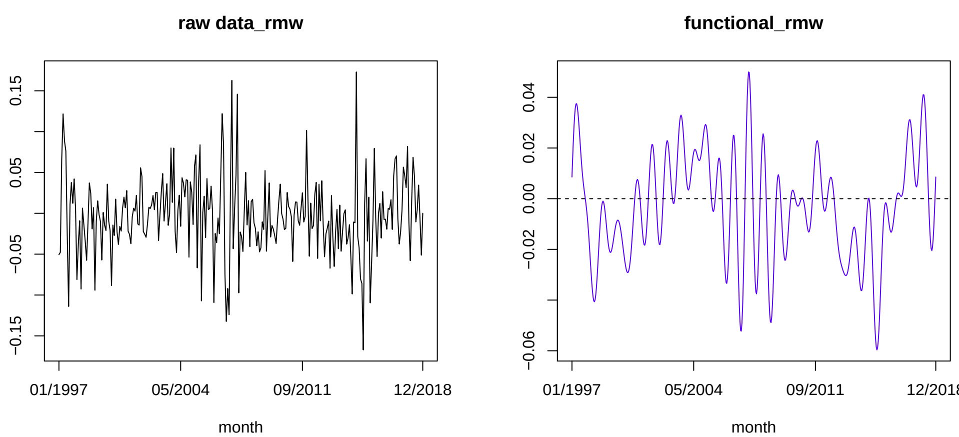

Figure 4: The profitability factor in the Chinese stock market (January 1997 - December 2018). The left panel shows raw data of the profitability factor, and the right panel shows the functional profitability factor.

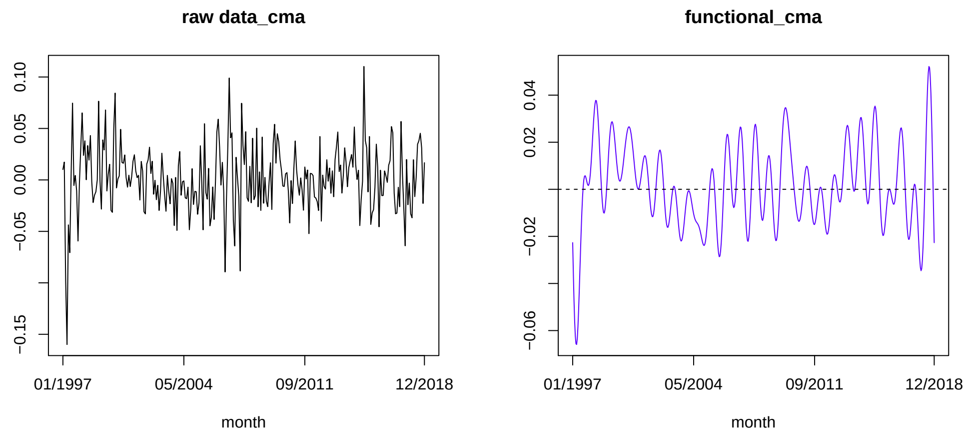

Figure 5: The investment factor in the Chinese stock market (January 1997 - December 2018). The left panel shows raw data of the investment factor, and the right panel shows the functional investment factor.

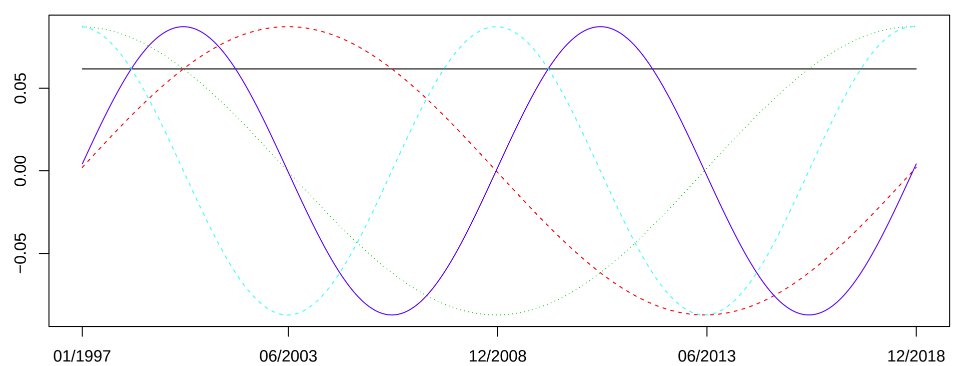

Figure 6: The Fourier basis functions for time-varying betas

5. Functional Factor Models

In this section, we want to investigate the time variation of each beta in the Chinese stock market. First, we construct 25 (or 100) Size-B/M portfolios [9] (B/M represents book-to-market ratio) with all A shares in order to get the value-weight (VW) excess returns. Then, we employ the functional factor models with 25 (or 100) Size-B/M portfolios for time-varying betas (see (25) and (26)). As mentioned earlier, one advantage of functional factor models is relaxing the constant-betas assumption. Now we point out another advantage: for every risk factor, functional factor models can obtain the conjoint $\beta_j(t)$ of all portfolios, which exhibits the common exposures of all A shares to each risk factor in the Chinese stock market. In section 5.2, we show that different portfolio constructions have little influence on the time variation of betas.

The least squares fitting criterion with 25 (or 100) portfolios is:

$$\min_{\{\beta_j(t)\}_{j=1}^{m}} \quad \sum_{i=1}^{25} \int_0^1 \left(R_i(t) - \sum_{j=1}^{m} \beta_j(t) \overline{F_{i,j}}(t) \right)^2 dt \tag{25}$$

or

$$\min_{\{\beta_j(t)\}_{j=1}^{m}} \quad \sum_{i=1}^{100} \int_0^1 \left(R_i(t) - \sum_{j=1}^{m} \beta_j(t) \overline{F_{i,j}}(t) \right)^2 dt \tag{26}$$

5.1. Time-varying Betas

With functional excess returns on 25 (or 100) Size-B/M portfolios and functional time-series risk factors, we can employ the functional CAPM, the functional three-factor model, and the functional five-factor model respectively. The time-varying betas are shown in Figure7-Figure12 (the solid line displays the estimated $\beta_j(t)$ and the dashed lines indicate 95% confidence intervals for $\beta_j(t)$), and the regression details of functional factor models are shown in Table1 and 2 (Table1 and 2 display the coefficient vectors $\boldsymbol{b}_j$ for each $\beta_j(t)$).

[9] The reason why we use excess returns of portfolios instead of individual stocks is that there is a lack of data in individual stocks on some trading days due to suspension and other reasons, and using excess returns of portfolios does not affect the results.

Figure7 (or Figure8) reveals the time-series exposures to the market factor. We can see that our portfolios have positive exposures to the market factor in the Chinese stock market, which means the market factor always has a positive effect on the returns of A shares. From 1997 to 2003, exposures to the market factor have been increasing, even more than 1. In the early 1990s, when the stock exchange was established, Chinese shareholders showed great enthusiasm for investment. Even if the Chinese government promulgated the securities law in 1997 to regulate the stock market, it failed to weaken the enthusiasm of investors. On June 29, 1999, the price-earnings ratio of Shenzhen and Shanghai Stock Exchanges reached about 48 times. However, most shareholders do not have the expertise corresponding to their investment behaviour, In addition, many state-owned enterprises with imperfect development were listed during this period, and stock market regulation was very inadequate. All of this raises market risk. During 2005-2006, the Chinese government implemented the non-tradable share reform to improve the governance of listed state-owned enterprises and solve the interests-conflict problem of shareholders in the A-share market. As the Chinese stock market matures, exposures to the market factor gradually decline. This situation continued until the financial crisis broke out. As Schlueter and Sievers (2014) point out business risks have an impact on market beta and the market beta increased again after the financial crisis broke out. After several ups and downs in the stock market, more and more investors choose value investing, and exposures to the market risk are no longer as high as they were at first.

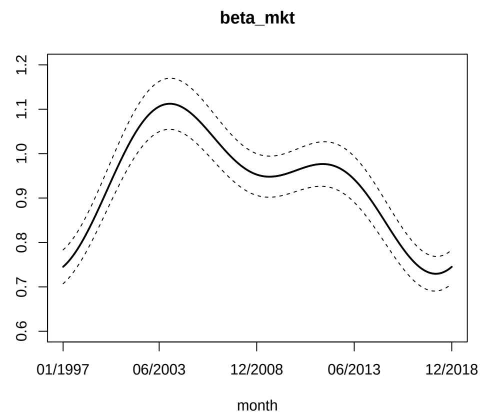

Figure 7: Time-varying beta of the functional CAPM with 25 Size-B/M portfolios

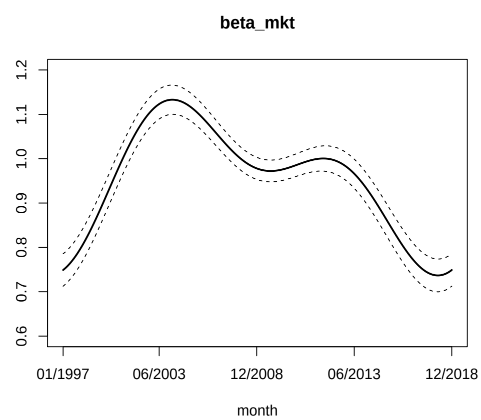

Figure 8: Time-varying beta of the functional CAPM with 100 Size-B/M portfolios

Figure9 (or Figure10) shows the time-series exposures to the market, size, and value factors. We conclude that: (a) Exposures to the market factor are always positive, which is the same as the functional CAPM. (b) Exposures to the size factor are positive in two periods (before June 1999 and after December 2006), but during 2000 to 2006, exposures to the size factor are negative, which may be due to non-tradable share structure in the Chinese stock market. Before 2005, there were two kinds of shares in Chinese listed companies: non-tradable shares and tradable shares. Non-tradable shares occupied a large proportion, which seriously hindered the circulation of listed companies shares, so the size effect of the listed company was hardly reflected in the price. After the non-tradable share reform, the securities market has become more market-oriented, and the small-scale companies have gradually shown higher excess returns. (c) Exposures to the value factor are only positive during 2011 to 2016, so the value factor has a positive effect on A shares only during a short period of time (from 2011 to 2016). Looking back at the history of the Chinese stock market, the common time-varying beta of the value factor may be related to the performance of the stock market. Before 2011, the stock market often experienced large or small bear markets and bull markets, making the market always in turmoil. During the financial crisis, exposures to the value factor reached its lowest point. Since the launch of stock index futures in 2011, the stock market has experienced a long period of stability, and the value factor began to have a positive effect on A shares, until June 2015 when the bear market reappeared, the common beta of the value factor changed. Therefore, the positive impact of the value factor may be related to stock market stability, and investors often pursue value investing when the stock market is stable.

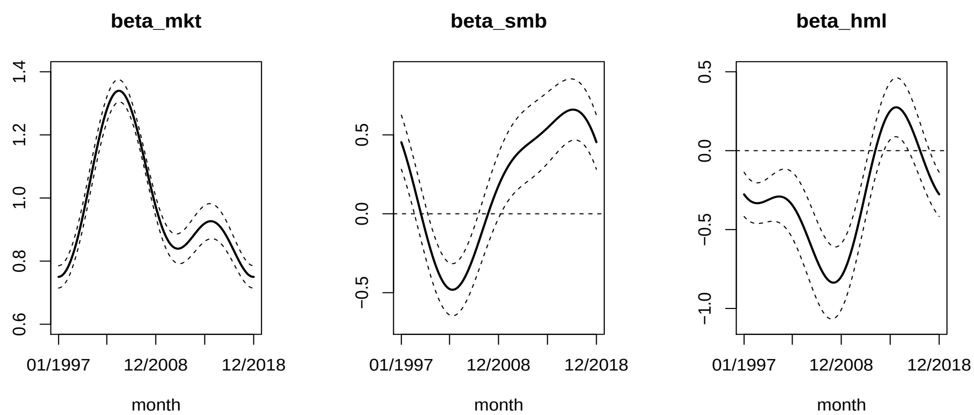

Figure 9: Time-varying beta of the functional 3-factor model with 25 Size-B/M portfolios

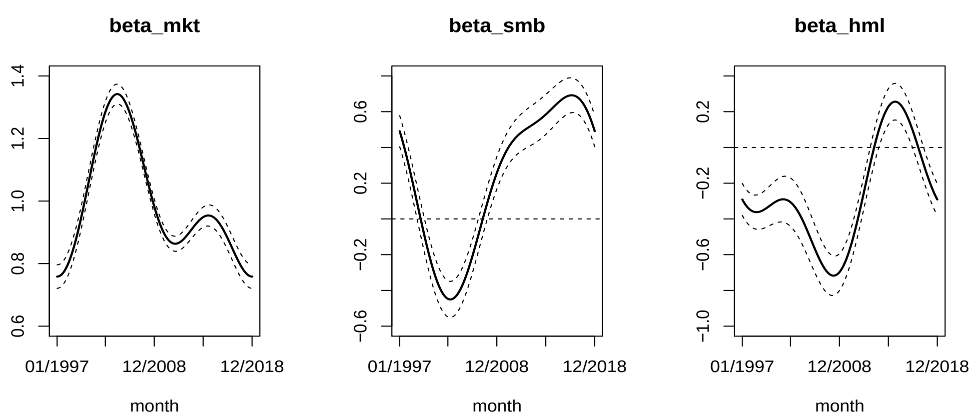

Figure 10: Time-varying beta of the functional 3-factor model with 100 Size-B/M portfolios

Figure11 (or Figure12) shows the time-series exposures to the market, size, value, profitability, and investment factors. Generally speaking, exposures to the market, size, and value factors in the functional five-factor model are similar to that of the functional three-factor model, and both the profitability factor and the investment factor have negative effects on A shares most of the time. To be specific: (a) The market factor has a positive effect on A shares at all times. (b) Combining the functional three-factor model, both models show the size factor has a positive effect from 2012 to 2016. Maybe it's because the Chinese stock market stayed in a relatively stable period after 2011, and became the bear market after June 2015. All in all, the size factor has a positive effect during a stable period. (c) Both the functional three-factor model and the functional five-factor model show that the value factor had a positive effect from 2011 to 2016. So the same as the size factor, the value factor has a positive effect in a stable period. (d) Exposures to the profitability factor were almost always negative in the whole sample period, and reached the minimum after the financial crisis. Valuation theory holds that the company's profitability is usually positively related to expected returns (Haugen et al., 1996; Fama and French, 2006). Nevertheless, portfolios have negative exposures to the profitability factor in the Chinese stock market, that is, most portfolios do not have high excess returns compared with the robust-profitability portfolio. (e) Exposures to the investment factor gradually changed from negative to positive when the non-tradable share reform was launched in China, and increased to 1 in 2007. The non-tradable share reform is conducive to improving corporate governance and investment structure. But during the financial crisis, exposures to the investment factor reached its maximum and began to decline due to stock market turbulence.

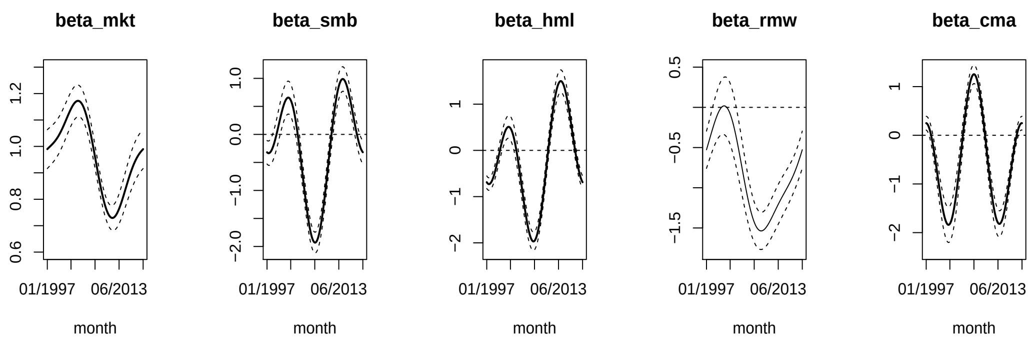

Figure 11: Time-varying beta of the functional 5-factor model with 25 Size-B/M portfolios

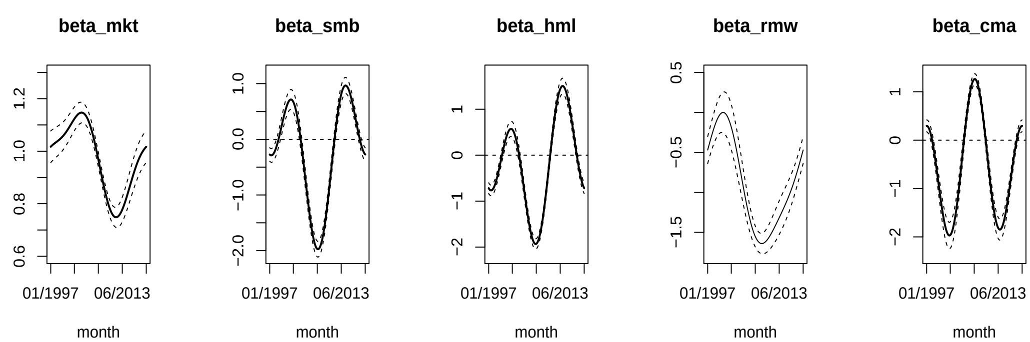

Figure 12: Time-varying beta of the functional 5-factor model with 100 Size-B/M portfolios

Table 1: Coefficient matrix $\hat{\boldsymbol{B}}$ with 25 value-weight Size-B/M portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$
Functional CAPM					
Market factor	15.1877	0.9243	-1.2103	0.1693	-1.0153
Functional Three-factor Model					
Market factor	15.9079	2.0642	-1.3010	-1.0733	-1.3455
Size factor	2.8540	-5.8313	1.6908	-0.2215	1.6446
Value factor	-4.7360	-3.3843	3.0480	-0.0041	-2.7960
Functional Five-factor Model					
Market factor	15.6311	2.1801	0.1384	-0.7365	0.1415
Size factor	-3.1744	-1.2970	9.2743	-1.5089	-10.5884
Value factor	-2.9267	-5.4290	7.3809	-1.0726	-13.0348
Profitability factor	-13.1955	6.4993	5.0407	0.3500	-1.9246
Investment factor	-8.5420	-0.1847	-5.7555	0.3540	14.6527

Table 2: Coefficient matrix $\hat{\boldsymbol{B}}$ with 100 value-weight Size-B/M portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$
Functional CAPM					
Market factor	15.4739	0.8829	-1.3317	0.1187	-1.0494
Functional Three-factor Model					
Market factor	16.1191	1.9201	-1.3112	-1.0403	-1.3865
Size factor	3.6129	-5.8935	1.4474	-0.1743	1.7797
Value factor	-4.3223	-3.0301	2.3910	-0.4082	-2.5844
Functional Five-factor Model					
Market factor	15.7536	1.9901	0.2493	-0.6252	-0.2562
Size factor	-3.1156	-0.6659	9.7549	-1.7854	-10.5950
Value factor	-2.6380	-4.9560	7.0438	-1.6655	-13.2370
Profitability factor	-14.3465	6.5660	6.0725	0.7583	-1.5171
Investment factor	-8.9860	-0.9261	-5.5226	1.2188	15.2399

5.2. The Robust Test

To answer whether different portfolio constructions with all A shares affect the time variation of betas, we perform functional factor models with 25 (or 100) Size-OP portfolios (OP represents operating profitability), 25 (or 100) Size-Inv portfolios (Inv represents investment) respectively. The robust tests show that different portfolio constructions have little influence on the time variation of betas. To be concise, the robust test results with 25 (or 100) Size-Inv portfolios are shown in Appendix A.

Figure13-Figure15 (or Figure16-Figure18)(the solid line displays the estimated $\beta_j(t)$ and the dashed lines indicate 95% confidence intervals for $\beta_j(t)$) and Table3 (or Table4) shows the functional regression results with 25 (or 100) Size-OP Portfolios, from which we can see the time variations of all $\beta_j(t)$ (t) for five risk factors are similar to that of the functional regressions with 25 (or 100) Size-B/M portfolios (specific differences are shown in Table3-Table4). Therefore, the common exposures of all portfolios to the risk factor are robust in the Chinese stock market.

25 Size-OP Portfolios

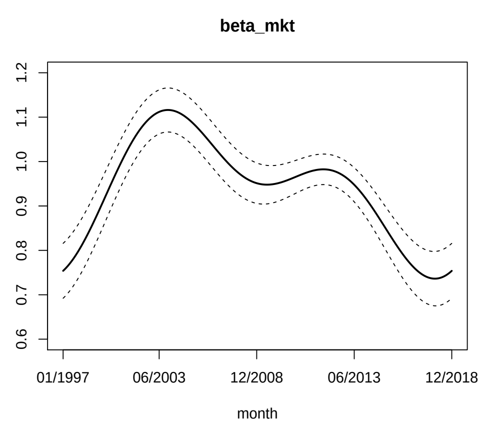

Figure 13: Time-varying beta of the functional CAPM with 25 Size-OP portfolios

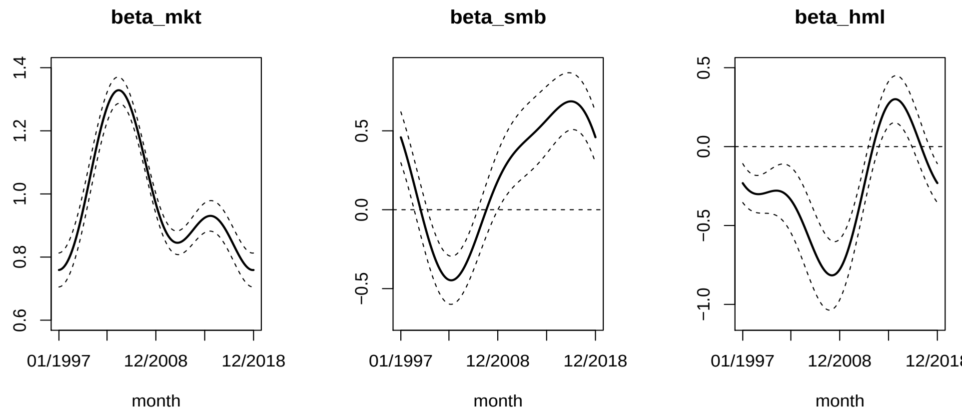

Figure 14: Time-varying beta of the functional 3-factor model with 25 Size-OP portfolios

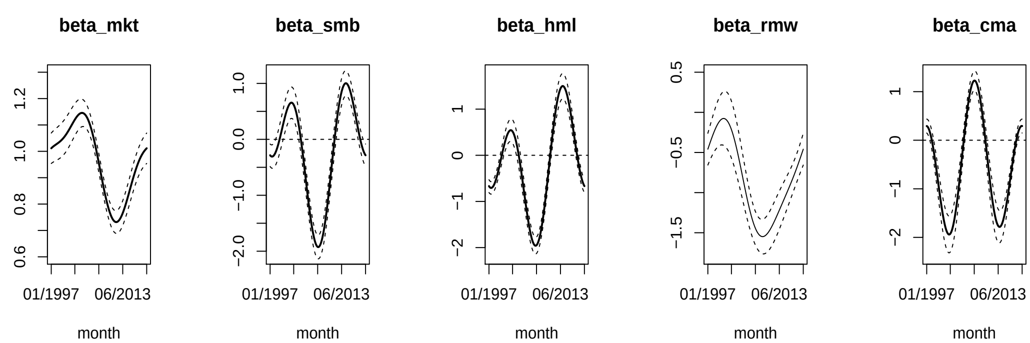

Figure 15: Time-varying beta of the functional 5-factor model with 25 Size-OP portfolios

Table 3: Coefficient matrix $\hat{\boldsymbol{B}}$ with 25 value-weight Size-OP portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$
Functional CAPM					
Market factor	15.2627	0.9248	-1.1516	0.2109	-1.0291
Functional Three-factor Model					
Market factor	15.9265	2.0043	-1.2286	-1.0055	-1.3308
Size factor	3.1283	-5.7862	1.6948	-0.3718	1.5159
Value factor	-4.3647	-3.4515	3.1996	-0.0853	-2.6831
Functional Five-factor Model					
Market factor	15.6219	2.0435	0.2878	-0.6781	0.2499
Size factor	-3.0152	-1.2114	9.4331	-1.8720	-10.4542
Value factor	-2.6679	-5.2267	7.4475	-1.1088	-13.0541
Profitability factor	-13.5796	5.8189	5.5413	0.2759	-1.3543
Investment factor	-8.7690	-0.9937	-5.3378	0.5203	14.9002

100 Size-OP Portfolios

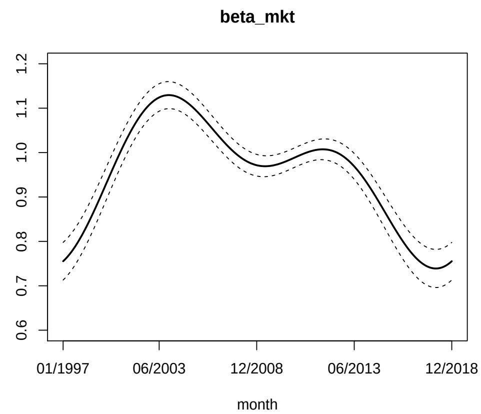

Figure 16: Time-varying beta of the functional CAPM with 100 Size-OP portfolios

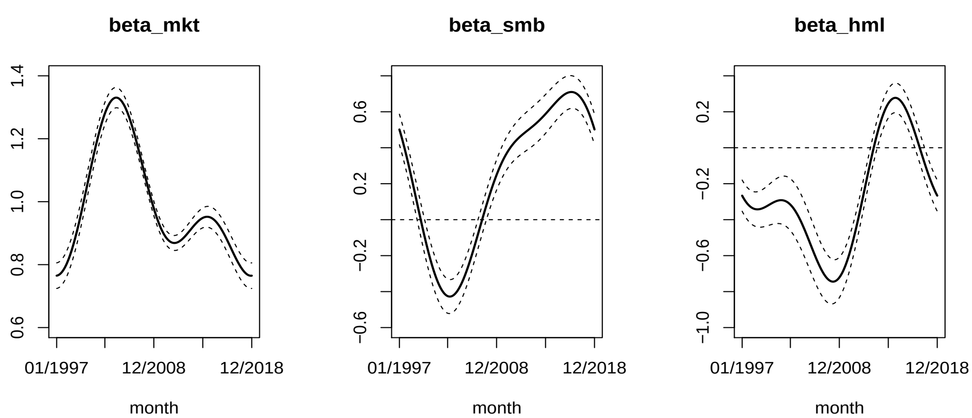

Figure 17: Time-varying beta of the functional 3-factor model with 100 Size-OP portfolios

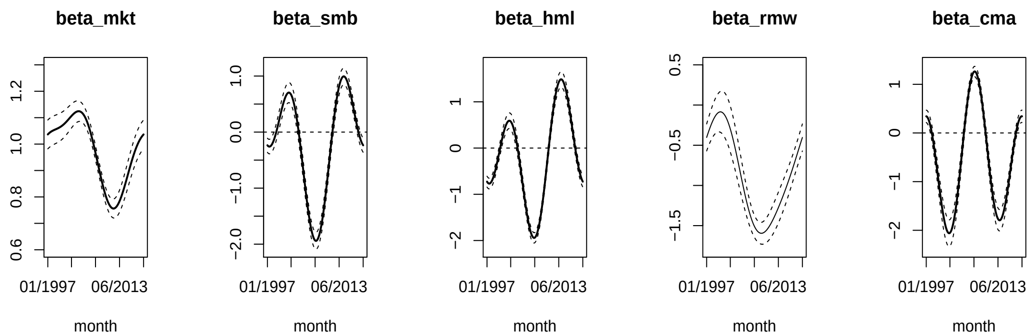

Figure 18: Time-varying beta of the functional 5-factor model with 100 Size-OP portfolios

Table 4: Coefficient matrix $\hat{\boldsymbol{B}}$ with 100 value-weight Size-OP portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$
Functional CAPM					
Market factor	15.4866	0.8723	-1.2587	0.2236	-1.0637
Functional Three-factor Model					
Market factor	16.1095	1.8975	-1.2486	-0.9398	-1.3731
Size factor	3.7256	-5.7859	1.5571	-0.3005	1.7158
Value factor	-4.2848	-3.1898	2.6713	-0.3640	-2.6040
Functional Five-factor Model					
Market factor	15.7804	1.8600	0.3746	-0.5502	0.3354
Size factor	-2.7861	-0.7330	9.7748	-2.1016	-10.3958
Value factor	-2.6966	-4.7765	7.0515	-1.5304	-13.2967
Profitability factor	-14.1769	5.6757	6.2191	0.5276	-0.9772
Investment factor	-8.9708	-1.7240	-5.2188	1.1588	15.4871

6. Conclusion

This paper introduces the functional factor models with time-varying betas (the functional CAPM, the functional three-factor model, and the functional five-factor model) and investigates the time variations of exposures to risk factors (the market, size, value, profitability, and investment factors). The advantage of functional factor models is that it figures out time-varying betas directly because the estimated betas of functional regression can be functional, so functional factor models modify the constant-beta assumption for the CAPM and the multi-factor models. The results show that in the Chinese stock market, (a) the market factor always has a positive effect on returns of A shares over the whole period. In the early years once the stock exchange was established, exposures to the market factor have been increasing, even more than 1. After the non-tradable share reform, exposures to the market factor began to decrease until the financial crisis broke out. (b) The size factor had a positive effect on returns of A shares from 2012 to 2016, which is a relatively long stable period in China. (c) Similar to the size factor, the value factor had a positive effect in a stable period (from 2011 to 2016). (d) The profitability factor always has a negative effect on returns of A shares. Exposures to the profitability factor reached the minimum after the financial crisis. (e) The investment factor had a positive effect after the non-tradable share reform and turned to a negative effect since the 2008 financial crisis.

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Appendix A.

25 Size-Inv Portfolios

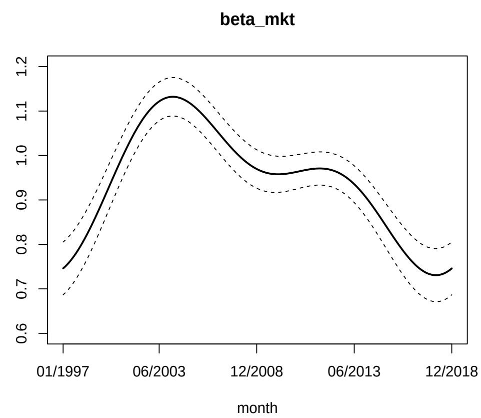

Figure A.19: Time-varying beta of the functional CAPM with 25 Size-Inv portfolios

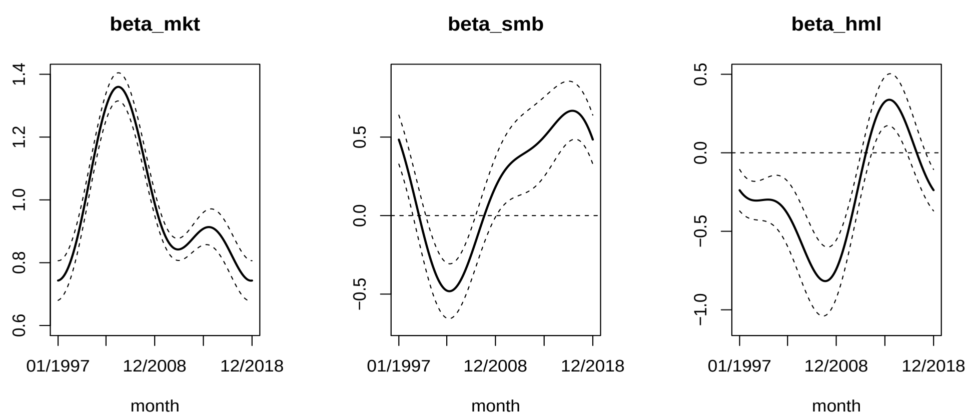

Figure A.20: Time-varying beta of the functional 3-factor model with 25 Size-Inv portfolios

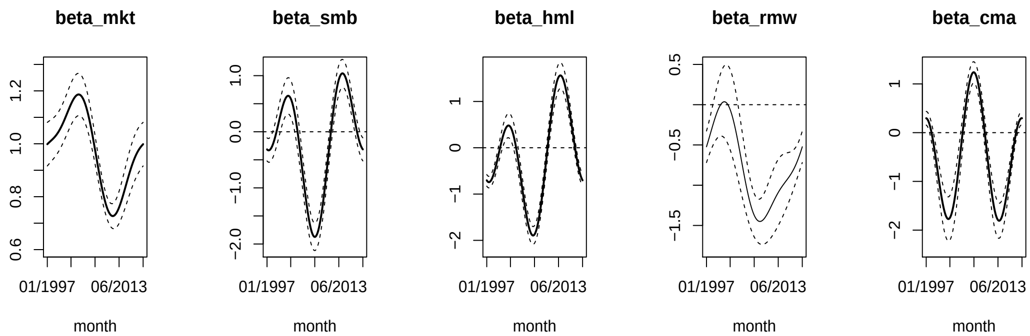

Figure A.21: Time-varying beta of the functional 5-factor model with 25 Size-Inv portfolios

Table A.5: Coefficient matrix $\hat{\boldsymbol{B}}$ with 25 value-weight Size-Inv portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$
Functional CAPM					
Market factor	15.2965	1.0508	-1.3031	0.0925	-0.9896
Functional Three-factor Model					
Market factor	15.9649	2.2212	-1.4356	-1.0749	-1.3346
Size factor	2.8167	-5.5896	1.8389	-0.3275	1.8716
Value factor	-4.2286	-4.0609	2.9558	0.3259	-2.6180
Functional Five-factor Model					
Market factor	15.7517	2.2476	0.0879	-0.7704	0.2078
Size factor	-2.7254	-1.8044	8.9829	-1.2806	-10.5627
Value factor	-2.4465	-6.0799	6.8165	-0.6723	-12.9895
Profitability factor	-12.4034	5.9513	4.7625	0.4820	-2.1523
Investment factor	-8.1057	0.1966	-5.4027	0.0222	14.5346

100 Size-Inv Portfolios

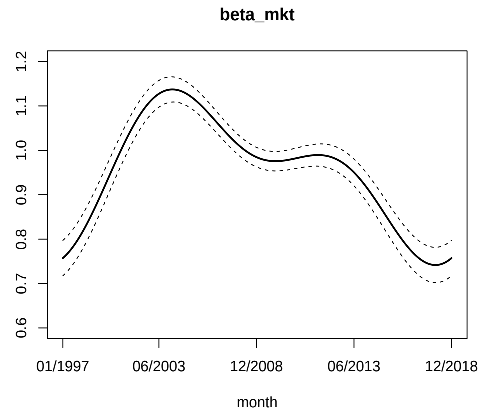

Figure A.22: Time-varying beta of the functional CAPM with 100 Size-Inv portfolios

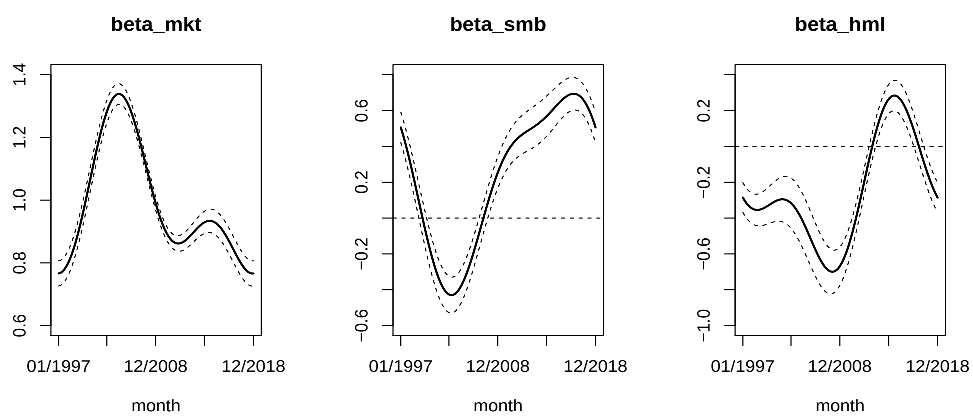

Figure A.23: Time-varying beta of the functional 3-factor model with 100 Size-Inv portfolios

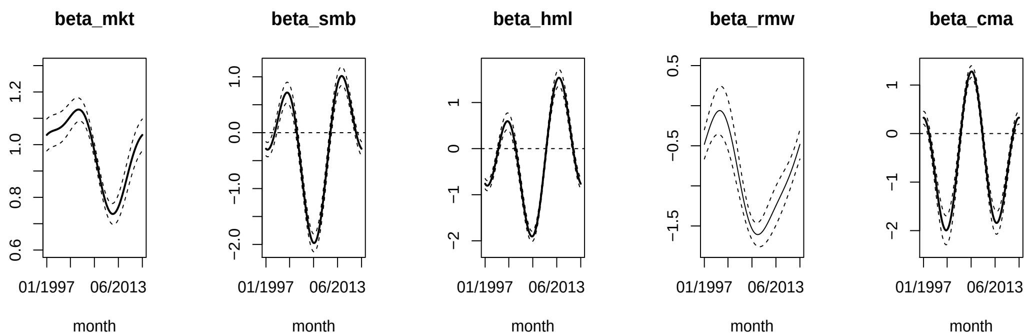

Figure A.24: Time-varying beta of the functional 5-factor model with 100 Size-Inv portfolios

Table A.6: Coefficient matrix $\hat{\boldsymbol{B}}$ with 100 value-weight Size-Inv portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$
Functional CAPM					
Market factor	15.4862	0.9980	-1.3236	0.1262	-0.9734
Functional Three-factor Model					
Market factor	16.0898	2.0172	-1.3053	-0.9860	-1.2866
Size factor	3.6786	-5.6800	1.5478	-0.2042	1.7981
Value factor	-4.0751	-3.3008	2.2550	-0.2346	-2.5596
Functional Five-factor Model					
Market factor	15.7404	1.9822	0.3208	-0.5988	0.4205
Size factor	-2.9324	-0.9960	9.6985	-1.6874	-10.8225
Value factor	-2.4168	-5.1033	6.6233	-1.4078	-13.4735
Profitability factor	-14.2424	5.8592	5.9251	0.6462	-1.5528
Investment factor	-8.8781	-1.0287	-5.4331	0.8587	15.4496